Hong Kong Mathematics Olympiad (1996 – 97)
Heat Event (Individual)
香港数学竞赛 (1996 – 97)
初赛项目 (个人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. Let n be a positive integer. If $n^2 = 29p + 1$, where p is a prime number, find the value of n.

   设 n 为一正整数。若 $n^2 = 29p + 1$， 其中 p 为质数，试求 n 之值。

2. If the width of a rectangle is increased by $\frac{1}{3}$ m, its area will be increased by $\frac{5}{3}$ m^2. If its length is decreased by $\frac{1}{2}$ m, its area will be decreased by $\frac{9}{5}$ m^2. Let the area of the rectangle be x m^2, find the value of x.

   若一长方形之阔度增加 $\frac{1}{3}$ 米，其面积增加 $\frac{5}{3}$ 平方米。若其长度减少 $\frac{1}{2}$ 米，则面积减少 $\frac{9}{5}$ 平方米。设该长方形之面积为 x 平方米，求 x 之值。

3. If s is the sum of all positive factors of 1234, find the value of s.

   若 s 为 1234 之所有正因子的总和，求 s 之值。

4. Let $x = \frac{1}{x}$, find the value of $\frac{x^2+2x-3}{x-1} \div \frac{x+5}{x^2+3x-6}$.

   设 $x = \frac{1}{x}$，求 $\frac{x^2+2x-3}{x-1} \div \frac{x+5}{x^2+3x-6}$ 的值。

5. Find the value of $1^2 - 2^2 + 3^2 - 4^2 + \cdots + 99^2 - 100^2$.

   求 $1^2 - 2^2 + 3^2 - 4^2 + \cdots + 99^2 - 100^2$ 的值。

6. If $yz : zx : xy = 1 : 2 : 3$, find $\frac{x}{yz} : \frac{y}{zx}$.

若 $yz : zx : xy = 1 : 2 : 3$，求 $\frac{x}{yz} : \frac{y}{zx}$。

7. Find the real roots of the equation :

$$x(x+1)(x^2+x+1) = x.$$

求下列方程的实根：

$$x(x+1)(x^2+x+1) = x。$$

8. There are 6 students in a class. Everyone sends one Christmas card to each of the rest of the class. Find the total number of cards sent out by the class.

某班有 6 位学生。每位学生送给班中其余各位同学一张圣诞咭，求该班学生寄出圣诞咭的总数。

9. If $2x^2 - 8x + k \equiv 2(x-2)^2 + 9$, find the value of k.

若 $2x^2 - 8x + k \equiv 2(x-2)^2 + 9$，求 k 之值。

10. If the ten-digit number $1357p1357p$ is divisible by 9, find the value of p.

若十位数 $1357p1357p$ 可被 9 整除，求 p 之值。